PAGE-71 (E.KREYSZIG) (8TH EDITION)

17-TO-20

Verify that the given functions form a basis of solutions of the given equation and solve the initial value problem.

17. $y'' + 9y = 0$ $\quad y(0) = 4,\ y'(0) = -6$ $\quad \cos 3x,\ \sin 3x$

Ans:- we have,

$$y'' + 9y = 0 \quad \text{---- (1)}$$

$$\Rightarrow \frac{d^2y}{dx^2} + 9y = 0$$

$$\Rightarrow (D^2 + 9)y = 0 \qquad \left(\because D = \frac{d}{dx}\right)$$

The auxiliary equation is

$$\lambda^2 + 9 = 0$$

$$\Rightarrow \lambda^2 = -9 = 9i^2$$

$$\Rightarrow \lambda = \pm 3i$$

Thus the general solution is

$$y = e^{0x}(C_1 \cos 3x + C_2 \sin 3x)$$

$$\Rightarrow y = C_1 \cos 3x + C_2 \sin 3x \quad \text{---- (2)}$$

So, $\cos 3x$ and $\sin 3x$ are the basis of solutions of equation (1)

$$y' = -3C_1 \sin 3x + 3C_2 \cos 3x \quad \text{---- (3)}$$

$y(0) = 4$ i.e $y = 4$ when $x = 0$ from equation (2)

$$4 = C_1 \cos 3\cdot 0 + C_2 \sin 3\cdot 0$$

$$\Rightarrow C_1 = 4 \quad \text{---- (4)}$$

$y'(0) = -6$ i.e $y' = -6$ when $x = 0$ from equation (3)

$$-6 = -3C_1 \sin 3\cdot 0 + 3C_2 \cos 3\cdot 0$$

$$\Rightarrow 3C_2 = -6 \Rightarrow C_2 = -2 \quad \text{---- (5)}$$

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Putting the value of C_1 and C_2 in equation (2) the solution is

$$y = 4\cos 3x - 2\sin 3x.$$

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NO-1-TO-4

Evaluate the following

1. D^2+3D, $\cosh 3x$, $e^{-x}+e^{2x}$, $10-e^{-3x}$

Ans:-

$(D^2+3D)\cosh 3x$

$= D^2\cosh 3x + 3D\cosh 3x$

$= \frac{d^2}{dx^2}\cosh 3x + 3\frac{d}{dx}\cosh 3x$

$= \frac{d}{dx}(3\sinh 3x) + 3(3\sinh 3x)$

$= 9\cosh 3x + 9\sinh 3x$

$(D^2+3D)(e^{-x}+e^{2x})$

$= D^2(e^{-x}+e^{2x}) + 3D(e^{-x}+e^{2x})$

$= \frac{d^2}{dx^2}(e^{-x}+e^{2x}) + 3\frac{d}{dx}(e^{-x}+e^{2x})$

$= \frac{d}{dx}(-e^{-x}+2e^{2x}) + 3(-e^{-x}+2e^{2x})$

$= e^{-x}+4e^{2x}-3e^{-x}+6e^{2x} = 10e^{2x}-2e^{-x}$

$(D^2+3D)(10-e^{-3x})$

$= D^2(10-e^{-3x}) + 3D(10-e^{-3x})$

$= \frac{d^2}{dx^2}(10-e^{-3x}) + 3\frac{d}{dx}(10-e^{-3x})$

$= \frac{d}{dx}(3e^{-3x}) + 3(3e^{-3x})$

$= -9e^{-3x} + 9e^{-3x}$

$= 0.$

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NO-1-TO-4

Verify the given function is a solution and derive the corresponding real general solution.

1. $y = c_1 e^{(1+i)x} + c_2 e^{(1-i)x}$ ——(1) $y'' - 2y' + 2y = 0$ ——(2)

Ans:- We have,

$$y = c_1 e^{(1+i)x} + c_2 e^{(1-i)x}$$

$$y' = (1+i) c_1 e^{(1+i)x} + (1-i) c_2 e^{(1-i)x}$$

$$y'' = (1+i)^2 c_1 e^{(1+i)x} + (1-i)^2 c_2 e^{(1-i)x}$$

Putting in equation (2)

LHS $y'' - 2y' + 2y$

$$= (1+i)^2 c_1 e^{(1+i)x} + (1-i)^2 c_2 e^{(1-i)x} - 2(1+i) c_1 e^{(1+i)x}$$
$$- 2(1-i) c_2 e^{(1-i)x} + 2 c_1 e^{(1+i)x} + 2 c_2 e^{(1-i)x}$$

$$= [(1+i)^2 - 2(1+i) + 2] c_1 e^{(1+i)x} + [(1-i)^2 - 2(1-i) + 2] c_2 e^{(1-i)x}$$

$$= [1 - 1 + 2i - 2 - 2i + 2] c_1 e^{(1+i)x} + [1 - 1 - 2i - 2 + 2i + 2] c_2 e^{(1-i)x}$$

$$= 0\, c_1 e^{(1+i)x} + 0\, c_2 e^{(1-i)x}$$

$$= 0 = RHS$$

So, equation (1) is a solution of equation (2)

Now $y = c_1 e^{(1+i)x} + c_2 e^{(1-i)x}$

$$= c_1 e^x e^{ix} + c_2 e^x e^{-ix}$$

$$= e^x [c_1 \cos x + i c_1 \sin x] + e^x [c_2 \cos x - i c_2 \sin x]$$

$$= e^x [(c_1 + c_2) \cos x + (i c_1 - i c_2) \sin x]$$

$$= e^x [A \cos x + B \sin x]$$ where, $A = c_1 + c_2$, $B = i(c_1 - c_2)$

is the required corresponding real general solution.

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NOTE:

9f atleast one of the function y_1 or y_2 is identically zero on an interval I then y_1 and y_2 are linearly independent on I. otherwise y_1 and y_2 are linearly dependent on I

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20. x^2, $x^2 \ln x$ $(x \geq 1)$

Let $y_1 = x^2$ $y_2 = x^2 \ln x$ I: $x \geq 1$

we have. $y_1 \equiv 0$ at $x = 0 \notin I$

$y_2 \equiv 0$ at $x = 1 \in I$

Thus y_1 and y_2 are linearly independent on I.

21. $\ln x$, $\ln(x^4)$ $(x > 1)$

Let $y_1 = \ln x$ $y_2 = \ln(x^4)$ I: $x > 1$

we have, $y_1 \equiv 0$ at $x = 1 \notin I$

$y_2 \equiv 0$ at $x = 1 \notin I$

Thus y_1 and y_2 are linearly dependent on I

25. $x|x|$, x^2 $(|x| \leq 1)$

Let $y_1 = x|x|$ $y_2 = x^2$ I: $-1 \leq x \leq 1$

we have, $y_1 \equiv 0$ at $x = 0 \in I$

$y_2 \equiv 0$ at $x = 0 \in I$

Thus, y_1 and y_2 are linearly independent on I.

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